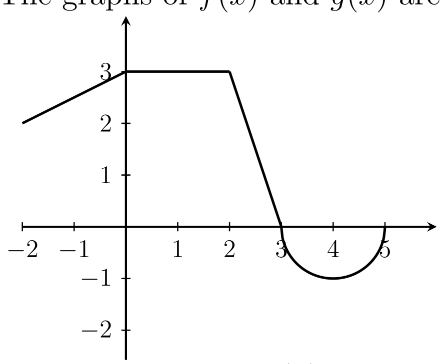
Objectives:

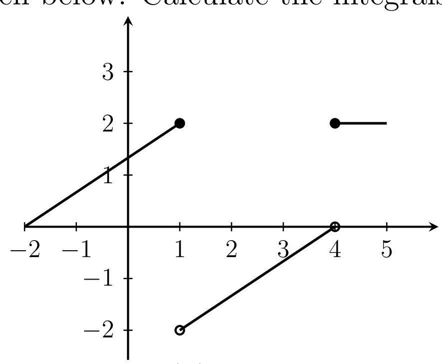
- Practice using properties of definite integrals.
- Compare values of definite integrals.
- Use antiderivatives to evaluate definite integrals.

1. $\displaystyle\int_0^{2\pi} (x + \sin(x))\, dx$

2. The graphs of $f(x)$ and $g(x)$ are given below. Calculate the integrals.



$f(x)$



$g(x)$

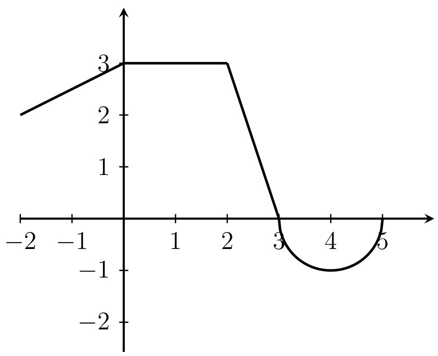
(a) $\int_{-2}^{1} f(x) + g(x)dx$

(b) $\int_{2}^{5} 10f(x)dx$=

(c) $\int_{-2}^{1} g(x) + 5dx$

(d) $\int_{2}^{1} g(x)dx + \int_{4}^{2} g(x)dx$

Comparing Integrals: For the function $f(x)$ in the previous problem, draw a function $h(x)$ on the axis such that $h(x) \geq f(x)$ for all x values in the interval $[0, 5]$:



How does $\int_0^5 h(x)$ compare to $\int_0^5 f(x)$?

In general we can say that if $f(x)$ ______ $h(x)$ for all x in the interval $[a, b]$, then $\int_a^b f(x)$ ______ $\int_a^b h(x)$.

In particular:

(1) If $f(x) \geq 0$ for all x in $[a, b]$:

(2) If $m \leq f(x) \leq M$ for all x in $[a, b]$ where m, M are constants:

Example: It would be very difficult to calculate $\int_{-2}^{3} \sin\left(\frac{1}{x}\right) dx$. However, we can compare the integral we want to know about to integrals that are easy to compute:

Evaluation Theorem (or, Fundamental Theorem of Calculus, Part II)

If f is ______ on ______

and F is any ______ , (i.e. $F'(x) =$ ______), then

We use the notation ______ to denote ______ .

Note:

Examples

1. $\displaystyle\int_{-1}^{2} x^4 dx$

2. $\displaystyle\int_{0}^{1} \frac{1}{1+x^2} dx$

3. $\displaystyle\int_{2}^{10} \left(e^x + 5x - \frac{1}{x}\right) dx$

Because of this relationship between the integral of $f(x)$ and the antiderivative of $f(x)$, we write ____________ to mean ________________________ . We call this expression an __________________ .

Note:

So now we have 3 ways of calculating an indefinite integral:

Interpreting the integral:
The Evaluation Theorem also appears as the ________________________ .

Since $F'(x) = f(x)$ is the ________________________ , the Evaluation Theorem tell us that

the ____ is equal to ____________ ,

which we call the ____________________________________ .

Examples:

If $f(x)$ represents:	Then $\int_a^b f(x)dx = F(b) - F(a)$ represents:
Velocity	
Marginal Cost	
Growth Rate of a Population	

Note: